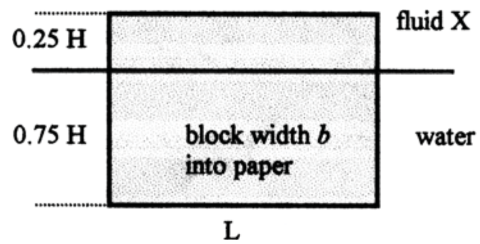
2.103 A solid block, of specific gravity 0.9, floats such that 75% of its volume is in water and 25% of its volume is in fluid X, which is layered above the water. What is the specific gravity of fluid X?



Solution: The block is sketched at right. A force balance is

$$0.9\gamma(\text{HbL}) = \gamma(0.75\text{HbL}) + \text{SG}_X\gamma(0.25\text{HbL})$$

$$0.9 - 0.75 = 0.25\text{SG}_X, \quad \boldsymbol{SG_X = 0.6} \quad \textit{Ans.}$$

2.104 The can in Fig. P2.104 floats in the position shown. What is its weight in newtons?

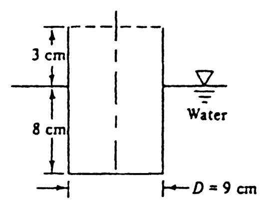


Fig. P2.104

Solution: The can weight simply equals the weight of the displaced water (neglecting the air above):

$$W = \gamma\upsilon_{\text{displaced}} = (9790)\frac{\pi}{4}(0.09\text{ m})^2(0.08\text{ m}) = \mathbf{5.0\ N} \quad \textit{Ans.}$$

2.105 Archimedes, when asked by King Hiero if the new crown was pure gold (SG = 19.3), found the crown weight in air to be 11.8 N and in water to be 10.9 N. Was it gold?

Solution: The buoyancy is the difference between air weight and underwater weight:

$$B = W_{\text{air}} - W_{\text{water}} = 11.8 - 10.9 = 0.9\text{ N} = \gamma_{\text{water}}\upsilon_{\text{crown}}$$

$$\text{But also} \quad W_{\text{air}} = (\text{SG})\gamma_{\text{water}}\upsilon_{\text{crown}}, \quad \text{so} \quad W_{\text{in water}} = B(\text{SG} - 1)$$

$$\text{Solve for} \quad \text{SG}_{\text{crown}} = 1 + W_{\text{in water}}/B = 1 + 10.9/0.9 = \mathbf{13.1\ (not\ pure\ gold)} \quad \textit{Ans.}$$

2.106 A spherical helium balloon is 2.5 m in diameter and has a total mass of 6.7 kg. When released into the U. S. Standard Atmosphere, at what altitude will it settle?

2.113 A *spar buoy* is a rod weighted to float vertically, as in Fig. P2.113. Let the buoy be maple wood (SG = 0.6), 2 in by 2 in by 10 ft, floating in seawater (SG = 1.025). How many pounds of steel (SG = 7.85) should be added at the bottom so that h = 18 in?

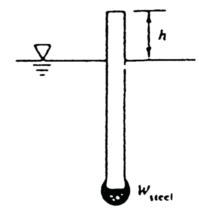


Fig. P2.113

Solution: The relevant volumes needed are

$$\text{Spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(10) = 0.278\ \text{ft}^3; \quad \text{Steel volume} = \frac{W_{steel}}{7.85(62.4)}$$

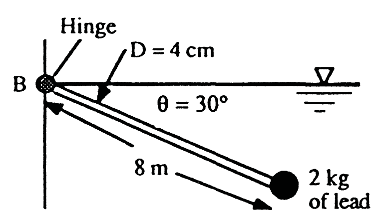
$$\text{Immersed spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(8.5) = 0.236\ \text{ft}^3$$

The vertical force balance is: buoyancy B = W_{wood} + W_{steel},

$$\text{or:} \quad 1.025(62.4)\left[0.236 + \frac{W_{steel}}{7.85(62.4)}\right] = 0.6(62.4)(0.278) + W_{steel}$$

$$\text{or:} \quad 15.09 + 0.1306W_{steel} = 10.40 + W_{steel}, \quad \text{solve for} \quad W_{steel} \approx \mathbf{5.4\ lbf} \quad \textit{Ans.}$$

2.114 The uniform rod in the figure is hinged at B and in static equilibrium when 2 kg of lead (SG = 11.4) are attached at its end. What is the specific gravity of the rod material? What is peculiar about the rest angle $\theta = 30°$?



Solution: First compute buoyancies: B_{rod} = 9790(π/4)(0.04)^2(8) = 98.42 N, and W_{lead} = 2(9.81) = 19.62 N, B_{lead} = 19.62/11.4 = 1.72 N. Sum moments about B:

$$\sum M_B = 0 = (SG - 1)(98.42)(4\cos 30°) + (19.62 - 1.72)(8\cos 30°) = 0$$

$$\text{Solve for} \quad \mathbf{SG_{rod} = 0.636} \quad \textit{Ans. (a)}$$

The angle θ drops out! The rod is neutrally stable for **any tilt angle!** *Ans.* (b)

2.120 A uniform wooden beam (SG = 0.65) is 10 cm by 10 cm by 3 m and hinged at A. At what angle will the beam float in 20°C water?

Solution: The total beam volume is $3(.1)^2 = 0.03\ m^3$, and therefore its weight is W = (0.65)(9790)(0.03) = 190.9 N, acting at the centroid, 1.5 m down from point A. Meanwhile, if the submerged length is H, the buoyancy is $B = (9790)(0.1)^2H = 97.9H$ newtons, acting at H/2 from the lower end. Sum moments about point A:

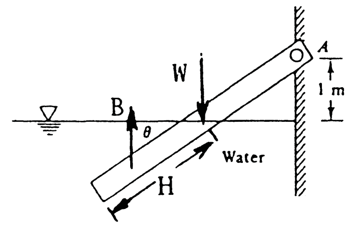


Fig. P2.120

$$\sum M_A = 0 = (97.9H)(3.0 - H/2)\cos\theta - 190.9(1.5\cos\theta),$$

$$\text{or:}\quad H(3 - H/2) = 2.925, \quad \text{solve for} \quad H \approx 1.225\ \text{m}$$

Geometry: 3 – H = 1.775 m is out of the water, or: $\sin\theta = 1.0/1.775$, or $\theta \approx$ **34.3°** *Ans.*

2.121 The uniform beam in the figure is of size L by h by b, with $b, h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

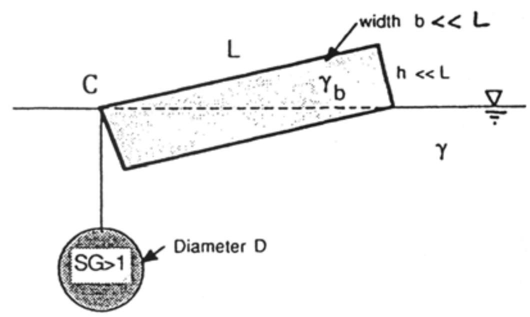


Fig. P2.121

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at L/3 from the left corner. Sum moments about the left corner, point C:

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. (a)}$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also}\quad T = (W - B)_{sphere} = (SG - 1)\gamma\frac{\pi}{6}D^3, \quad \text{so that} \quad D = \left[\frac{\mathbf{Lhb}}{\boldsymbol{\pi}\mathbf{(SG - 1)}}\right]^{\mathbf{1/3}} \quad \textit{Ans. (b)}$$

Solution: First, how high is the container? Well, 1 fluid oz. = 1.805 in^3, hence 12 fl. oz. = 21.66 in^3 = $\pi(1.5\text{ in})^2h$, or $h \approx 3.06$ *in*—It is a fat, nearly square little glass. Second, determine the acceleration toward the center of the merry-go-round, noting that the angular velocity is $\Omega = (12\text{ rev/min})(1\text{ min/60 s})(2\pi\text{ rad/rev}) = 1.26$ rad/s. Then, for $r = 4$ ft,

$$a_x = \Omega^2 r = (1.26\text{ rad/s})^2(4\text{ ft}) = 6.32\text{ ft/s}^2$$

Then, for steady rotation, the water surface in the glass will slope at the angle

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{6.32}{32.2 + 0} = 0.196, \quad \text{or:} \quad \Delta h_{\text{left to center}} = (0.196)(1.5\text{ in}) = 0.294\text{ in}$$

Thus the glass should be filled to no more than 3.06 − 0.294 ≈ 2.77 inches
This amount of liquid is $\upsilon = \pi(1.5\text{ in})^2(2.77\text{ in}) = 19.6\text{ in}^3 \approx$ **10.8 fluid oz.** *Ans.*

2.139 The tank of liquid in the figure P2.139 accelerates to the right with the fluid in rigid-body motion. (a) Compute a_x in m/s^2. (b) Why doesn't the solution to part (a) depend upon fluid density? (c) Compute gage pressure at point A if the fluid is glycerin at 20°C.

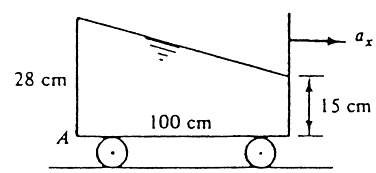


Fig. P2.139

Solution: (a) The slope of the liquid gives us the acceleration:

$$\mathbf{tan}\,\theta = \frac{a_x}{g} = \frac{28 - 15\text{ cm}}{100\text{ cm}} = 0.13, \quad \text{or:} \quad \theta = 7.4°$$

$$\textit{thus} \quad a_x = 0.13g = 0.13(9.81) = \mathbf{1.28\ m/s^2} \quad \textit{Ans.}\text{ (a)}$$

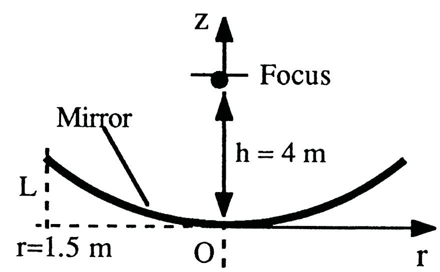
(b) Clearly, the solution to (a) is purely geometric and does not involve fluid density. *Ans.* (b)
(c) From Table A-3 for glycerin, $\rho = 1260\text{ kg/m}^3$. There are many ways to compute p_A. For example, we can go straight down on the left side, using only gravity:

$$p_A = \rho g \Delta z = (1260\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.28\text{ m}) = \mathbf{3460\ Pa\ (gage)} \quad \textit{Ans.}\text{ (c)}$$

Or we can start on the right side, go down 15 cm with g and across 100 cm with a_x:

$$p_A = \rho g \Delta z + \rho a_x \Delta x = (1260)(9.81)(0.15) + (1260)(1.28)(1.00)$$
$$= 1854 + 1607 = \mathbf{3460\ Pa} \quad \textit{Ans.}\text{ (c)}$$

2.158* It is desired to make a 3-m-diameter parabolic telescope mirror by rotating molten glass in rigid-body motion until the desired shape is achieved and then cooling the glass to a solid. The focus of the mirror is to be 4 m from the mirror, measured along the centerline. What is the proper mirror rotation rate, in rev/min?



Solution: We have to review our math book, or Mark's Manual, to recall that the *focus* F of a parabola is the point for which all points on the parabola are equidistant from both the focus and a so-called "directrix" line (which is one focal length below the mirror).
For the focal length h and the z-r axes shown in the figure, the equation of the parabola is given by $\mathbf{r^2 = 4hz}$, with $h = 4$ m for our example.
Meanwhile the equation of the free-surface of the liquid is given by $\mathbf{z = r^2\Omega^2/(2g)}$.
Set these two equal to find the proper rotation rate:

$$z = \frac{r^2\Omega^2}{2g} = \frac{r^2}{4h}, \quad \text{or:} \quad \Omega^2 = \frac{g}{2h} = \frac{9.81}{2(4)} = 1.226$$

$$\textit{Thus} \quad \Omega = 1.107\ \frac{\text{rad}}{\text{s}}\left(\frac{60}{2\pi}\right) = \mathbf{10.6\ rev/min} \quad \textit{Ans.}$$

The focal point F is far above the mirror itself. If we put in $r = 1.5$ m and calculate the mirror depth "L" shown in the figure, we get L ≈ 14 centimeters.

2.159 The three-legged manometer in Fig. P2.159 is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity Ω about the central tube, (a) derive a formula to find the change of height in the tubes; (b) find the height in cm in each tube if Ω = 120 rev/min. [*HINT*: The central tube must supply water to *both* the outer legs.]

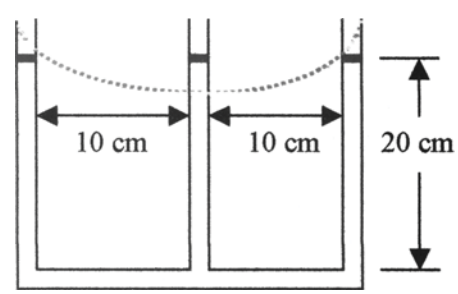


Fig. P2.159

Solution: (a) The free-surface during rotation is visualized as the **red** dashed line in Fig. P2.159. The outer right and left legs experience an increase which is one-half that of the central leg, or $\Delta h_O = \Delta h_C/2$. The total displacement between outer and center menisci is, from Eq. (2.64) and Fig. 2.23, equal to $\Omega^2R^2/(2g)$. The center meniscus falls two-thirds of this amount and feeds the outer tubes, which each rise one-third of this amount above the rest position:

$$\boldsymbol{\Delta h_{outer} = \frac{1}{3}\Delta h_{total} = \frac{\Omega^2 R^2}{6g}} \qquad \boldsymbol{\Delta h_{center} = -\frac{2}{3}\Delta h_{total} = -\frac{\Omega^2 R^2}{3g}} \quad \textit{Ans.}\text{ (a)}$$

For the particular case $R = 10$ cm and $\Omega = 120$ r/min = (120)(2π/60) = 12.57 rad/s, we obtain

$$\frac{\Omega^2 R^2}{2g} = \frac{(12.57\text{ rad/s})^2(0.1\text{ m})^2}{2(9.81\text{ m/s}^2)} = 0.0805\text{ m};$$

$$\Delta h_O \approx \mathbf{0.027\ m}\ (\textit{up}) \quad \Delta h_C \approx \mathbf{-0.054\ m}\ (\textit{down}) \quad \textit{Ans.}\text{ (b)}$$

Therefore the handle force required is $F = P/16 = 222/16 \approx \mathbf{14\ lbf}$ *Ans.*

2.21 In Fig. P2.21 all fluids are at 20°C. Gage A reads 350 kPa absolute. Determine (a) the height h in cm; and (b) the reading of gage B in kPa absolute.

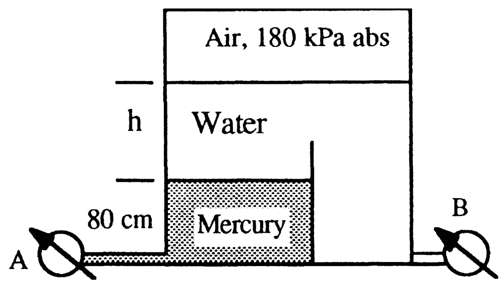


Fig. P2.21

Solution: Apply the hydrostatic formula from the air to gage A:

$$p_A = p_{air} + \sum \gamma h$$

$$= 180000 + (9790)h + 133100(0.8) = 350000 \text{ Pa},$$

Solve for $h \approx \mathbf{6.49\ m}$ *Ans.* (a)

Then, with *h* known, we can evaluate the pressure at gage B:

$$p_B = 180000 + 9790(6.49 + 0.80) = 251000 \text{ Pa} \approx \mathbf{251\ kPa} \quad \textit{Ans.}\text{ (b)}$$

2.22 The fuel gage for an auto gas tank reads proportional to the bottom gage pressure as in Fig. P2.22. If the tank accidentally contains 2 cm of water plus gasoline, how many centimeters "h" of air remain when the gage reads "full" in error?

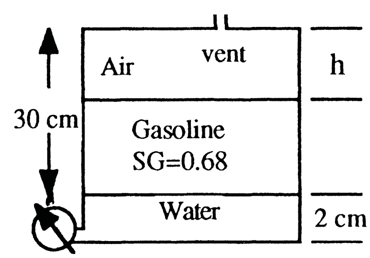


Fig. P2.22

Solution: Given $\gamma_{gasoline} = 0.68(9790) = 6657\ N/m^3$, compute the pressure when "full":

$$p_{full} = \gamma_{gasoline}(\text{full height}) = (6657\ N/m^3)(0.30\ m) = 1997\ Pa$$

Set this pressure equal to 2 cm of water plus "Y" centimeters of gasoline:

$$p_{full} = 1997 = 9790(0.02\ m) + 6657Y, \quad \text{or} \quad Y \approx 0.2706\ m = 27.06\ cm$$

Therefore the air gap h = 30 cm − 2 cm(water) − 27.06 cm(gasoline) ≈ **0.94 cm** *Ans.*

2.23 In Fig. P2.23 both fluids are at 20°C. If surface tension effects are negligible, what is the density of the oil, in kg/m^3?

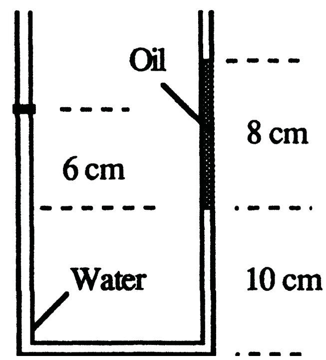


Fig. P2.23

Solution: Move around the U-tube from left atmosphere to right atmosphere:

$$p_a + (9790\ N/m^3)(0.06\ m) - \gamma_{oil}(0.08\ m) = p_a,$$

$$\text{solve for } \gamma_{oil} \approx 7343\ N/m^3,$$

$$\text{or: } \rho_{oil} = 7343/9.81 \approx \mathbf{748\ kg/m^3} \quad \textit{Ans.}$$

2.24 In Prob. 1.2 we made a crude integration of atmospheric density from Table A.6 and found that the atmospheric mass is approximately $m \approx$ 6.08E18 kg. Can this result be used to estimate sea-level pressure? Can sea-level pressure be used to estimate m?

Solution: Yes, atmospheric pressure is essentially a result of the weight of the air above. Therefore the air weight divided by the surface area of the earth equals sea-level pressure:

$$p_{sea\text{-}level} = \frac{W_{air}}{A_{earth}} = \frac{m_{air}g}{4\pi R_{earth}^2} \approx \frac{(6.08E18\ kg)(9.81\ m/s^2)}{4\pi(6.377E6\ m)^2} \approx \mathbf{117000\ Pa} \quad \textit{Ans.}$$

This is a little off, thus our mass estimate must have been a little off. If global average sea-level pressure is actually 101350 Pa, then the mass of atmospheric air must be more nearly

$$m_{air} = \frac{A_{earth}p_{sea\text{-}level}}{g} \approx \frac{4\pi(6.377E6\ m)^2(101350\ Pa)}{9.81\ m/s^2} \approx \mathbf{5.28E18\ kg} \quad \textit{Ans.}$$

2.35 Water flows upward in a pipe slanted at 30°, as in Fig. P2.35. The mercury manometer reads h = 12 cm. What is the pressure difference between points (1) and (2) in the pipe?

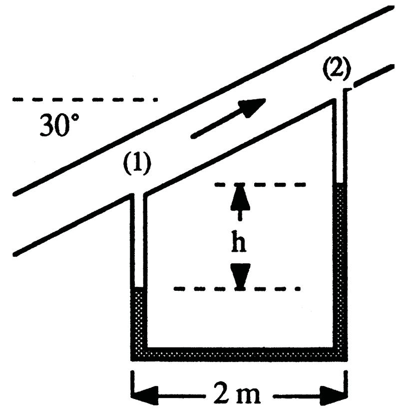


Fig. P2.35

Solution: The vertical distance between points 1 and 2 equals (2.0 m)tan 30° or 1.155 m. Go around the U-tube hydrostatically from point 1 to point 2:

$$p_1 + 9790h - 133100h - 9790(1.155\text{ m}) = p_2,$$

$$\text{or:}\quad p_1 - p_2 = (133100 - 9790)(0.12) + 11300 = \mathbf{26100\ Pa} \quad \textit{Ans.}$$

2.36 In Fig. P2.36 both the tank and the slanted tube are open to the atmosphere. If L = 2.13 m, what is the angle of tilt ϕ of the tube?

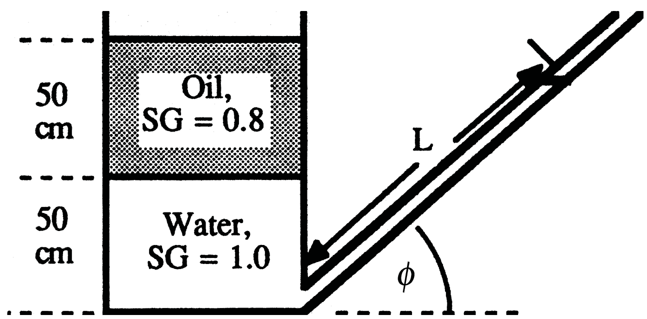


Fig. P2.36

Solution: Proceed hydrostatically from the oil surface to the slanted tube surface:

$$p_a + 0.8(9790)(0.5) + 9790(0.5) - 9790(2.13\sin\phi) = p_a,$$

$$\text{or:}\quad \sin\phi = \frac{8811}{20853} = 0.4225, \quad \text{solve}\ \ \phi \approx \mathbf{25°} \quad \textit{Ans.}$$

2.37 The inclined manometer in Fig. P2.37 contains Meriam red oil, SG = 0.827. Assume the reservoir is very large. If the inclined arm has graduations 1 inch apart, what should θ be if each graduation represents 1 psf of the pressure pA?

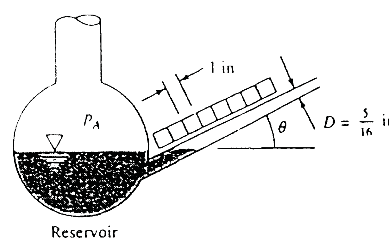


Fig. P2.37

2.44 Water flows downward in a pipe at 45°, as shown in Fig. P2.44. The mercury manometer reads a 6-in height. The pressure drop p2 − p1 is partly due to friction and partly due to gravity. Determine the total pressure drop and also the part due to friction only. Which part does the manometer read? Why?

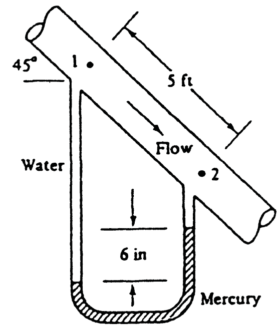


Fig. P2.44

Solution: Let "h" be the distance down from point 2 to the mercury-water interface in the right leg. Write the hydrostatic formula from 1 to 2:

$$p_1 + 62.4\left(5\sin 45° + h + \frac{6}{12}\right) - 846\left(\frac{6}{12}\right) - 62.4h = p_2,$$

$$p_1 - p_2 = \underbrace{(846 - 62.4)(6/12)}_{\text{....friction loss...}} - \underbrace{62.4(5\sin 45°)}_{\text{..gravity head..}} = 392 - 221$$

$$= \mathbf{171\,\frac{lbf}{ft^2}} \quad \textit{Ans.}$$

The manometer reads only the *friction loss* of 392 lbf/ft^2, not the gravity head of 221 psf.

2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than Patmosphere?

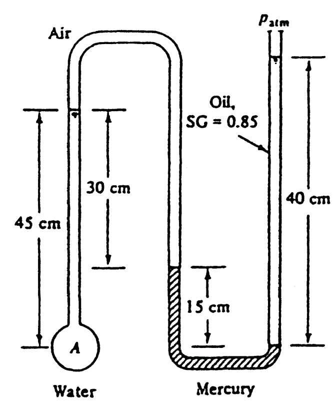
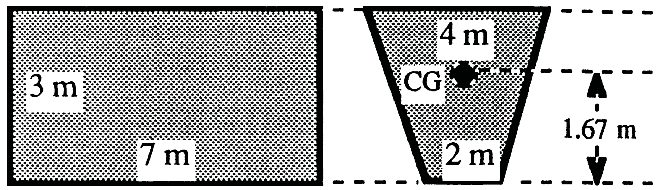


Fig. P2.45

Solution: Take γ = 9790 N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$p_{atm} + (0.85)(9790)(0.4\text{ m}) - (133100)(0.15\text{ m}) - (12)(0.30\text{ m}) + (9790)(0.45\text{ m}) = p_A,$$

or: $p_A = p_{atm} - 12200\text{ Pa} = \mathbf{12200\ Pa\ (vacuum)}$ *Ans.*



Solution: (a) The total volume of oil in the vat is (3 m)(7 m)(4 m + 2 m)/2 = 63 m^3. Therefore the weight of oil in the vat is

$$W = \gamma_{oil}(\text{Vol}) = (0.85)(9790 \text{ N/m}^3)(63 \text{ m}^3) = \mathbf{524,000 \text{ N}} \quad \textit{Ans.} \text{ (a)}$$

(b) The force on the horizontal bottom surface of the vat is

$$F_{bottom} = \gamma_{oil} h_{CG} A_{bottom} = (0.85)(9790)(3 \text{ m})(2 \text{ m})(7 \text{ m}) = \mathbf{350,000 \text{ N}} \quad \textit{Ans.} \text{ (b)}$$

Note that F is <u>less</u> than the total weight of oil—the student might explain why they differ? (c) I found in my statics book that the centroid of this trapezoid is 1.33 m below the surface, or 1.67 m above the bottom, as shown. Therefore the side-panel force is

$$F_{side} = \gamma_{oil} h_{CG} A_{side} = (0.85)(9790)(1.33 \text{ m})(9 \text{ m}^2) = \mathbf{100,000 \text{ N}} \quad \textit{Ans.} \text{ (c)}$$

These are large forces. Big vats have to be strong!

2.51 Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force F on the gate and its center of pressure position *X*.

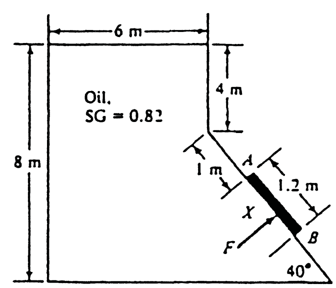


Fig. P2.51

Solution: The centroidal depth of the gate is

$$h_{CG} = 4.0 + (1.0 + 0.6)\sin 40° = 5.028 \text{ m},$$

$$\text{hence} \quad F_{AB} = \gamma_{oil} h_{CG} A_{gate} = (0.82 \times 9790)(5.028)(1.2 \times 0.8) = \mathbf{38750 \text{ N}} \quad \textit{Ans.}$$

The line of action of F is slightly below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(0.8)(1.2)^3\sin 40°}{(5.028)(1.2 \times 0.8)} = -0.0153 \text{ m}$$

Thus the position of the center of pressure is at $X = 0.6 + 0.0153 \approx \mathbf{0.615 \text{ m}}$ *Ans.*

Solution: (a) The resultant force F, may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG}A = (9790\ \text{N/m}^3)(3+1.5\ \text{m})(5\ \text{m}\times 2\ \text{m}) = 440{,}550\ \text{N} = \mathbf{441\ kN} \quad \textit{Ans.}\ (a)$$

(b) The horizontal force acts as though BC were vertical, thus h_{CG} is halfway down from C and acts on the projected area of BC.

$$F_H = (9790)(4.5)(3\times 2) = 264{,}330\ \text{N} = \mathbf{264\ kN} \quad \textit{Ans.}\ (b)$$

The vertical force is equal to the weight of fluid above BC,

$$F_V = (9790)[(3)(4)+(1/2)(4)(3)](2) = 352{,}440 = \mathbf{352\ kN} \quad \textit{Ans.}\ (b)$$

The resultant is the same as part (a): $F = [(264)^2 + (352)^2]^{1/2}$ **= 441 kN.**

2.58 In Fig. P2.58, weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level *h* will dislodge the gate?

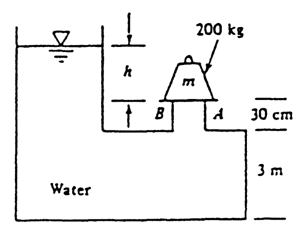


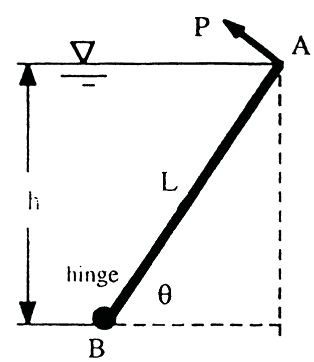
Fig. P2.58

Solution: The centroidal depth is exactly equal to *h* and force F will be upward on the gate. Dislodging occurs when F equals the weight:

$$F = \gamma h_{CG}A_{gate} = (9790\ \text{N/m}^3)\ h\frac{\pi}{4}(0.8\ \text{m})^2 = W = (200)(9.81)\ \text{N}$$

$$\text{Solve for} \quad h = \mathbf{0.40\ m} \quad \textit{Ans.}$$

2.59 Gate AB has length L, width b into the paper, is hinged at B, and has negligible weight. The liquid level h remains at the top of the gate for any angle θ. Find an analytic expression for the force P, perpendicular to AB, required to keep the gate in equilibrium.



Solution: The centroid of the gate remains at distance L/2 from A and depth h/2 below the surface. For any θ, then, the hydrostatic force is $F = \gamma(h/2)Lb$. The moment of inertia of the gate is $(1/12)bL^3$, hence $y_{CP} = -(1/12)bL^3\sin\theta/[(h/2)Lb]$, and the center of pressure is $(L/2 - y_{CP})$ from point B. Summing moments about hinge B yields

$$PL = F(L/2 - y_{CP}), \quad \text{or:} \quad \mathbf{P = (\gamma hb/4)(L - L^2\sin\theta/3h)} \quad \textit{Ans.}$$

P2.60 In 1960, Auguste and Jacques Picard's self-propelled bathyscaphe *Trieste* set a record by descending to a depth of 35,800 feet in the Pacific Ocean, near Guam. The passenger sphere was 7 ft in diameter, 6 inches thick, and had a window 16 inches in diameter. (*a*) Estimate the hydrostatic force on the window at that depth. (*b*) If the window is vertical, how far below its center is the center of pressure?

Solution: At the surface, the density of seawater is about 1025 kg/m^3 (1.99 slug/ft^3). Atmospheric pressure is about 2116 lbf/ft^2. We could use these values, or estimate from Eq. (1.19) that the density at depth would be about 4.6% more, or 2.08 slug/ft^3. We could average these two to 2.035 slug/ft^3. The pressure at that depth would thus be approximately

$$p = p_a + \rho_{avg}\, g\, h = 2116 + (2.035\frac{slug}{ft^3})(32.2\frac{ft}{s^2})(35800\, ft) \approx 2{,}350{,}000\ \frac{lbf}{ft^2}$$

(*a*) This pressure, times the area of the 16-inch window, gives the desired force.

$$F_{window} = p_{CG}\, A = (2350000\frac{lbf}{ft^2})[\frac{\pi}{4}(\frac{16}{12}ft)^2] = \mathbf{3{,}280{,}000\ \ lbf} \qquad \textit{Ans.}(a)$$

Quite a lot of force, but the bathyscaphe was well designed.

(*b*) The distance down to the center of pressure on the window follows from Eq. (2.27):

$$y_{CP} = -\gamma\sin\theta\frac{I_{xx}}{F} = -[2.035*32.2\frac{lbf}{ft^3}]\sin(90^\circ)\frac{(\pi/4)(8/12\, ft)^4}{3280000\, lbf} = \mathbf{-3.2E-6\ \ ft.} \quad \textit{Ans.}(b)$$

The center of pressure at this depth is only 38 micro inches below the center of the window.

2.61 Gate AB in Fig. P2.61 is a homogeneous mass of 180 kg, 1.2 m wide into the paper, resting on smooth bottom B. All fluids are at 20°C. For what water depth *h* will the force at point B be zero?

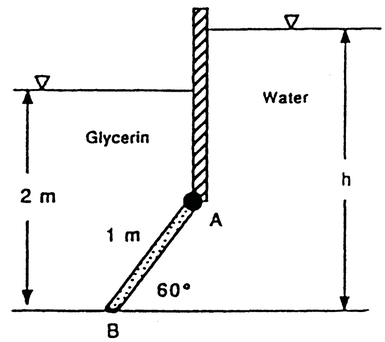


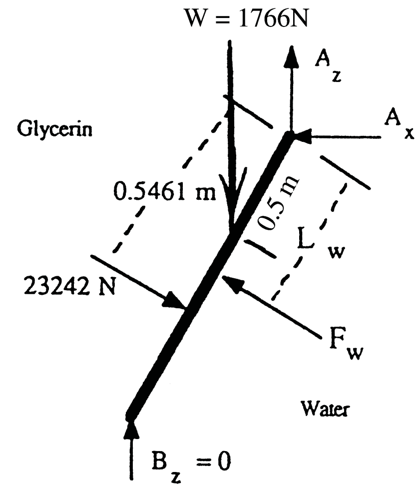
Fig. P2.61

Solution: Let γ = 12360 N/m^3 for glycerin and 9790 N/m^3 for water. The centroid of

AB is 0.433 m vertically below A, so hCG = 2.0 − 0.433 = 1.567 m, and we may compute the glycerin force and its line of action:

$$F_g = \gamma \bar{h} A = (12360)(1.567)(1.2) = 23242 \text{ N}$$

$$y_{CP,g} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{(1.567)(1.2)} = -0.0461 \text{ m}$$



These are shown on the freebody at right. The water force and its line of action are shown without numbers, because they depend upon the centroidal depth on the water side:

$$F_w = (9790)h_{CG}(1.2)$$

$$y_{CP} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{h_{CG}(1.2)} = -\frac{0.0722}{h_{CG}}$$

The weight of the gate, W = 180(9.81) = 1766 N, acts at the centroid, as shown above. Since the force at B equals zero, we may sum moments counterclockwise about A to find the water depth:

$$\sum M_A = 0 = (23242)(0.5461) + (1766)(0.5 \cos 60°) - (9790)h_{CG}(1.2)(0.5 + 0.0722/h_{CG})$$

Solve for $h_{CG,water} = 2.09$ m, or: $h = h_{CG} + 0.433 = \mathbf{2.52\ m}$ *Ans.*

2.62 Gate AB in Fig. P2.62 is 15 ft long and 8 ft wide into the paper, hinged at B with a stop at A. The gate is 1-in-thick steel, SG = 7.85. Compute the 20°C water level *h* for which the gate will start to fall.

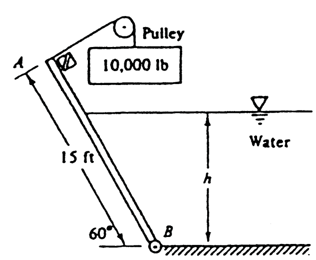


Fig. P2.62

Solution: Only the length (h csc 60°) of the gate lies below the water. Only this part

2.84 Determine (a) the total hydrostatic force on curved surface AB in Fig. P2.84 and (b) its line of action. Neglect atmospheric pressure and assume unit width into the paper.

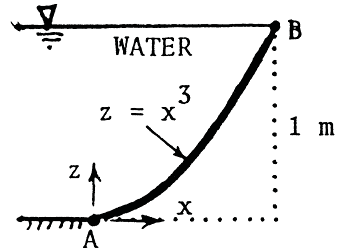


Fig. P2.84

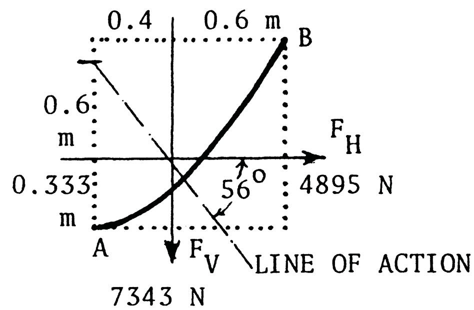
Solution: The horizontal force is

$$F_H = \gamma h_{CG} A_{vert} = (9790\ \text{N/m}^3)(0.5\ \text{m})(1 \times 1\ \text{m}^2) = 4895\ \text{N} \quad \text{at } 0.667\ \text{m below B.}$$

For the cubic-shaped surface AB, the weight of water above is computed by integration:

$$F_V = \gamma b \int_0^1 (1 - x^3)\,dx = \frac{3}{4}\gamma b$$

$$= (3/4)(9790)(1.0) = 7343\ \text{N}$$



The line of action (water centroid) of the vertical force also has to be found by integration:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^1 x(1 - x^3)\,dx}{\int_0^1 (1 - x^3)\,dx} = \frac{3/10}{3/4} = 0.4\ \text{m}$$

The vertical force of 7343 N thus acts at 0.4 m to the right of point A, or 0.6 m to the left of B, as shown in the sketch above. The resultant hydrostatic force then is

$$F_{total} = [(4895)^2 + (7343)^2]^{1/2} = \mathbf{8825\ N} \quad \text{acting at } \mathbf{56.31°} \text{ down and to the right.} \quad \textit{Ans.}$$

This result is shown in the sketch at above right. The line of action of F strikes the vertical above point A at 0.933 m above A, or 0.067 m below the water surface.

2.85 Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. P2.85.

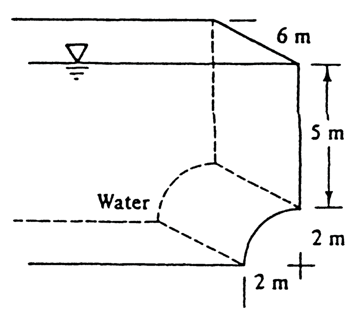


Fig. P2.85

Solution: The horizontal component is

$$F_H = \gamma h_{CG} A_{vert} = (9790)(6)(2 \times 6)$$
$$= \mathbf{705000\ N} \quad \textit{Ans.}\ \text{(a)}$$

P2.90 The tank in Fig. P2.90 is 120 cm long into the paper. Determine the horizontal and vertical hydrostatic forces on the quarter-circle panel AB. The fluid is water at 20°C. Neglect atmospheric pressure.

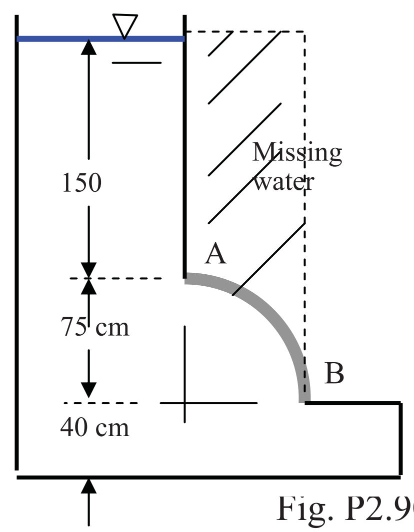


Fig. P2.90

Solution: For water at 20°C, take $\gamma = 9790\ \mathrm{N/m^3}$.

The vertical force on AB is the weight of the missing water above AB – see the dashed lines in Fig. P2.90. Calculate this as a rectangle plus a square-minus-a-quarter-circle:

$$\text{Missing water} = (1.5m)(0.75m)(1.2m) + (1-\pi/4)(0.75m)^2 = 2.16 + 0.145 = 2.305\, m^3$$

$$F_V = \gamma \upsilon = (9790\, N/m^3)(2.305\, m^3) = \mathbf{22{,}600\,N} \quad (\textit{vertical force})$$

The horizontal force is calculated from the vertical projection of panel AB:

$$F_H = p_{CG}\, h\, A_{projection} = (9790\frac{N}{m^3})(1.5 + \frac{0.75}{2}m)(0.75m)(1.2m) = \mathbf{16{,}500\,N} \quad (\textit{horizontal force})$$

2.91 The hemispherical dome in Fig. P2.91 weighs 30 kN and is filled with water and attached to the floor by six equally-spaced bolts. What is the force in each bolt required to hold the dome down?

Solution: Assuming no leakage, the hydrostatic force required equals the *weight of missing water*, that is, the water in a 4-m-diameter cylinder, 6 m high, minus the hemisphere and the small pipe:

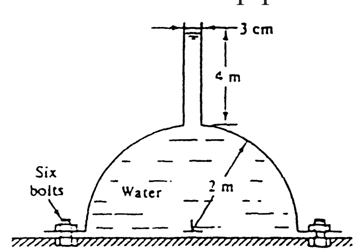


Fig. P2.91

$$
\begin{aligned}
F_{total} &= W_{2\text{-m-cylinder}} - W_{2\text{-m-hemisphere}} - W_{3\text{-cm-pipe}} \\
&= (9790)\pi(2)^2(6) - (9790)(2\pi/3)(2)^3 - (9790)(\pi/4)(0.03)^2(4) \\
&= 738149 - 164033 - 28 = 574088 \text{ N}
\end{aligned}
$$

The dome material helps with 30 kN of weight, thus the bolts must supply 574088–30000 or 544088 N. The force in each of 6 bolts is 544088/6 or $F_{bolt} \approx$ **90700 N** *Ans.*

2.92 A 4-m-diameter water tank consists of two half-cylinders, each weighing 4.5 kN/m, bolted together as in Fig. P2.92. If the end caps are neglected, compute the force in each bolt.

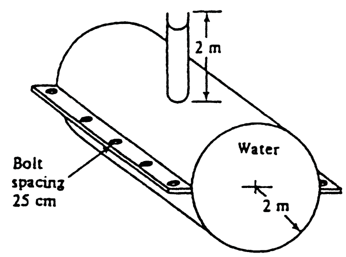


Fig. P2.92

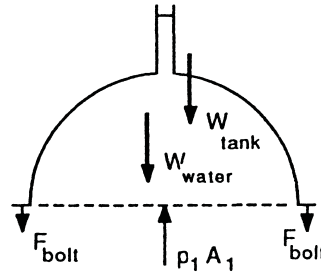
Solution: Consider a 25-cm width of upper cylinder, as at right. The water pressure in the bolt plane is

$$p_1 = \gamma h = (9790)(4) = 39160 \text{ Pa}$$

Then summation of vertical forces on this 25-cm-wide freebody gives

$$\sum F_z = 0 = p_1A_1 - W_{water} - W_{tank} - 2F_{bolt}$$

$$= (39160)(4\times 0.25) - (9790)(\pi/2)(2)^2(0.25)$$
$$-(4500)/4 - 2F_{bolt},$$



Solve for $F_{\text{one bolt}} = \mathbf{11300\ N}$ *Ans.*

2.93 In Fig. P2.93 a one-quadrant spherical shell of radius R is submerged in liquid of specific weight γ and depth h > R. Derive an analytic expression for the hydrodynamic force F on the shell and its line of action.

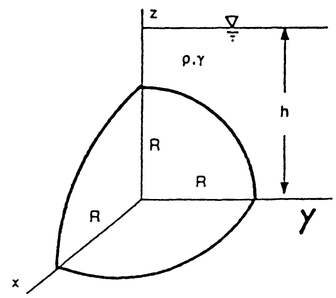


Fig. P2.93

Solution: The two horizontal components are identical in magnitude and equal to the force on the quarter-circle side panels, whose centroids are ($4R/3\pi$) above the bottom:

$$\text{Horizontal components:}\quad F_x = F_y = \gamma h_{CG}A_{vert} = \gamma\left(h - \frac{4R}{3\pi}\right)\frac{\pi}{4}R^2$$

Similarly, the vertical component is the weight of the fluid above the spherical surface:

$$F_z = W_{cylinder} - W_{sphere} = \gamma\left(\frac{\pi}{4}R^2h\right) - \gamma\left(\frac{1}{8}\frac{4}{3}\pi R^3\right) = \gamma\frac{\pi}{4}R^2\left(h - \frac{2R}{3}\right)$$

There is no need to find the (complicated) centers of pressure for these three components, for we know that the resultant on a spherical surface <u>*must pass through the center*</u>. Thus

$$F = \left[F_x^2 + F_y^2 + F_z^2\right]^{1/2} = \boldsymbol{\gamma\frac{\pi}{4}R^2\left[(h - 2R/3)^2 + 2(h - 4R/3\pi)^2\right]^{1/2}} \quad \textit{Ans.}$$

2.94 The 4-ft-diameter log (SG = 0.80) in Fig. P2.94 is 8 ft long into the paper and dams water as shown. Compute the net vertical and horizontal reactions at point C.

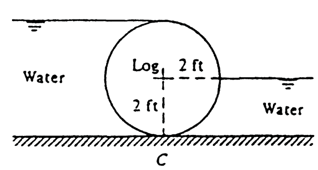
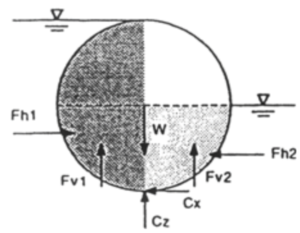


Fig. P2.94

Solution: With respect to the sketch at right, the horizontal components of hydrostatic force are given by



$$F_{h1} = (62.4)(2)(4 \times 8) = 3994 \text{ lbf}$$

$$F_{h2} = (62.4)(1)(2 \times 8) = 998 \text{ lbf}$$

The vertical components of hydrostatic force equal the weight of water in the shaded areas:

$$F_{v1} = (62.4)\frac{\pi}{2}(2)^2(8) = 3137 \text{ lbf}$$

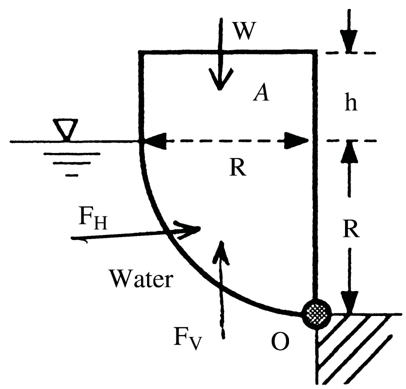
$$F_{v2} = (62.4)\frac{\pi}{4}(2)^2(8) = 1568 \text{ lbf}$$

The weight of the log is $W_{log} = (0.8 \times 62.4)\pi(2)^2(8) = 5018$ lbf. Then the reactions at C are found by summation of forces on the log freebody:

$$\sum F_x = 0 = 3994 - 998 - C_x, \quad \text{or} \quad C_x = \mathbf{2996\ lbf} \quad \textit{Ans.}$$

$$\sum F_z = 0 = C_z - 5018 + 3137 + 1568, \quad \text{or} \quad C_z = \mathbf{313\ lbf} \quad \textit{Ans.}$$

2.95 The uniform body A in the figure has width *b* into the paper and is in static equilibrium when pivoted about hinge O. What is the specific gravity of this body when (a) h = 0; and (b) h = R?



Solution: The water causes a horizontal and a vertical force on the body, as shown:

$$F_H = \gamma \frac{R}{2} Rb \quad at \quad \frac{R}{3} \ above\ O,$$

$$F_V = \gamma \frac{\pi}{4} R^2 b \quad at \quad \frac{4R}{3\pi} \ to\ the\ left\ of\ O$$

These must balance the moment of the body weight W about O:

$$\sum M_O = \frac{\gamma R^2 b}{2}\left(\frac{R}{3}\right) + \frac{\gamma \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \frac{\gamma_s \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \gamma_s Rhb\left(\frac{R}{2}\right) = 0$$